

# A C++ Code to Solve the DGLAP Equations Applied to Ultra High Energy Cosmic Rays

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## Abstract

We solve numerically the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations for the evolution of fragmentation functions using the Laguerre method. We extend this method to include supersymmetric evolution. The solution to the DGLAP equations is particularly interesting to calculate the expected spectra of Ultra High Energy Cosmic Rays in models where they are produced by the decay of a massive particle  $X$ ,  $M_X > 10^{12}$  GeV.

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# PROGRAM SUMMARY

*Title of program:* evolve

*Computer and operating system:* Program tested on Sun running SunOS 5.7, Alpha running OSF1 4.0, Dell-PC running Linux Mandrake 7.2.

*Programming language used:* C++ with g++ compiler

*No. Lines in distributed program:* 2500

*Keywords:* UHECR, Fragmentation Functions, DGLAP equations, Laguerre method, supersymmetry.

*Nature of Physical Problem:* To predict the spectra of UHECR produced by the decay of a dark matter superheavy particle with mass  $M_X$  one needs to calculate fragmentation functions at the energy scale  $M_X$ . These fragmentation functions at the scale  $M_X$  can be calculated from measured low energy fragmentation functions using the DGLAP equations.

*Method of solution:* The DGLAP equations are solved by expanding them in Laguerre polynomials which reduces their integration to the computation of a set of coefficients. These coefficients are given by algebraic recursive relations.

*Typical running time:* A few seconds.

*Restriction of the program:* Gluon coherence at  $x < 10^{-3}$  is not included.

## I. INTRODUCTION

Over one hundred cosmic ray events with an energy higher than  $4 \times 10^{19}$  eV have been observed by different observatories (see [1] for a recent review). Were their sources at cosmological distances,  $d > 50 - 100$  Mpc, they would interact with the Cosmic Microwave Background (CMB) on their way to the Earth and lose their enormous energy. Therefore, one expects the sources for these Ultra High Energy Cosmic Rays (UHECR) to be not far from the Galaxy. However, there are a few astrophysical sites in the galactic neighbourhood which could accelerate a charged particle to such high energy. Faced with this conundrum it has been suggested that UHECR are not accelerated at all but are created at this ultra high energy by the decay of massive dark matter particles  $X$  clustered in the galactic halo. In order to test this hypothesis one needs to calculate the angular distribution of events [2] and the spectra [3–6] produced by the decay of  $X$ . In the present work we will concentrate on the calculation of the spectra.

The  $X$  decay contribution to UHECR is proportional to the inclusive decay width of particle  $X$ , with mass  $M_X$ , into particle  $h$  [6]

$$\frac{1}{\Gamma_X} \frac{d\Gamma(X \rightarrow h + \dots)}{dx} = \sum_a \int_x^1 \frac{dz}{z} \frac{1}{\Gamma_a} \frac{d\Gamma_a(y, \mu^2, M_X^2)}{dy} \Big|_{y=x/z} D_a^h(z, \mu^2). \quad (1)$$

Here  $x$  is the fraction of the energy of  $X$  carried by  $h$  and  $z$  is the fraction of the energy of parton  $a$  carried by  $h$ . The first factor in the integrand, the decay width of  $X$  into parton  $a$ ,  $d\Gamma_a/dy$ , is calculable in perturbation theory. It encapsulates all kinematical effects in many-body decay. In lowest order and for two-body decay it is proportional to  $\delta(1 - y)$ . The second factor, the non-perturbative  $D_a^h$ , is the fragmentation function (FF) for particles of type  $h$  from partons of type  $a$ . The mass scale  $\mu$  is the factorization scale,  $\mu \sim M_X$  [7]. Particle  $h$  is any final state:  $n$ ,  $p$ ,  $\gamma$ ,  $e$ ,  $\nu_e$ ,  $\nu_\mu$  or  $\nu_\tau$ .

The FF satisfy a set of coupled integro-differential equations, the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [8,9]. Given experimental data at some low energy scale, say  $M_Z$ , an initial set of FF  $D_a^h(x, M_Z^2)$  can be extracted and evolved using the DGLAP equations, to obtain the FF at some higher scale  $D_a^h(x, M_X^2)$ .

Hadronic structure functions (SF) satisfy as well DGLAP equations. Although DGLAP equations for SF are similar to DGLAP equations for FF they are not equivalent. Several approaches have been taken to solve the DGLAP equations, usually applied to SF. One is the Mellin transform method [10] which transforms the integro-differential equations into ordinary differential equations. However, at the end one needs to invert the Mellin transform to find the solution in terms of  $x$  which is a process with notorious numerical problems. The QCDNUM program [11] defines a grid in  $x$  and the energy scale  $\mu^2$ . The calculation of the SF on the grid points is based on the computation of convolution integrals that are evaluated as weighted sums.

A very elegant method to solve the DGLAP equations was introduced by Furmański and Petronzio [12]. It expands these integro-differential equations using Laguerre polynomial so that their integration is reduced to calculating a set of numerical coefficients using simple algebraic recursive relations. The Laguerre method has been implemented in numerical codes by several groups [13–15].

Usually numerical codes deal with SF, and all of them are oriented to collider physics. Several include polarization and other aspects which add complexity to the numerical evolution and that are irrelevant for cosmic ray studies. Motivated as well by present or past experiments, most of the numerical codes evolve SF using Standard Model equations, without considering supersymmetry (SUSY) or any model beyond the Standard Model (SM). The theoretical basis of SUSY DGLAP evolution for SF was studied in Refs. [16,17]. Numerical solutions for the evolution of SF in several supersymmetric scenarios have recently been presented in Ref. [18]. In UHECR studies, including SUSY is of paramount importance. Most of the evolution from the low energy scale  $M_Z$  to the high energy scale  $M_X$  will be governed by SUSY equations as long as SUSY is a symmetry of nature and the SUSY breaking scale is of the order of the weak scale.

We have written a numerical code to evolve FF using the DGLAP equations. It has been written bearing in mind its application to UHECR and therefore does not include physics aspects such as spin dependent functions that are relevant for collider physics but not for UHECR physics. It includes SM evolution and SUSY evolution. We have chosen the Laguerre method to solve numerically the DGLAP equations. We have generalised the Laguerre method to include SUSY evolution, which is not a trivial task, at least in the way that this method is presented in the literature.

Matrix algebra becomes very important when solving SUSY DGLAP evolution by the Laguerre method. Therefore, it is convenient to code the algorithm using an object oriented language. We have chosen C++, which in the present context provides a good framework to perform matrix calculations. The use of templates available in this programming language simplifies the code. The final result is a fast algorithm with good accuracy for the relevant range of  $x$  and  $\mu^2$ .

## II. DGLAP EQUATIONS

### A. General Equations, Leading Order and Notation

The DGLAP equations can be written as

$$\frac{\partial D_a^h(x, \mu^2)}{\partial \ln \mu^2} = \sum_b \frac{\alpha_s(\mu^2)}{2\pi} P_{ba}(x, \alpha_s(\mu^2)) \otimes D_b^h(x, \mu^2), \quad (2)$$

where  $\alpha_s(\mu^2)$  is the strong coupling constant and  $P_{ba}(x, \alpha_s)$  is the splitting function for the parton branching  $a \rightarrow b$ . Here the convolution of two functions  $A(x)$  and  $B(x)$  is defined as

$$A(x) \otimes B(x) \equiv \int_x^1 \frac{dz}{z} A(z) B\left(\frac{x}{z}\right). \quad (3)$$

The splitting functions can be expanded perturbatively

$$P_{ba}(x, \alpha_s) = P_{ba}(x) + O(\alpha_s) \quad (4)$$

We will limit our study to leading order in  $\alpha_s$  and therefore ignore  $O(\alpha_s)$  corrections to the splitting functions.

It is also convenient to define the following dimensionless evolution parameter

$$\tau \equiv \frac{1}{2\pi b} \ln \frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)}, \quad (5)$$

$b$  being the coefficient in the leading order beta function governing the running of the strong coupling:  $\beta(\alpha_s) = -b\alpha_s^2$ . We take  $D_a^h$  to represent the sum of particle  $h$  and, if different, its antiparticle  $\bar{h}$ .

## B. Standard Model Equations

The Standard Model DGLAP equations for the evolution of fragmentations functions are well-known [7,19]. There are two parton species: quarks  $q_k$ ,  $k = 1, \dots, n_F$  and gluons  $g$ , with  $n_F$  the total number of flavours. Conventionally, one defines the following linear combinations (for ease of notation the superscript  $h$  is omitted)

$$D_{q_k^+} \equiv D_{q_k} + D_{\bar{q}_k} \quad (6)$$

$$D_q \equiv \sum_k D_{q_k^+} \quad (7)$$

$$D_{q_k^-} \equiv D_{q_k} - D_{\bar{q}_k} \quad (8)$$

$$D_{Q_k} \equiv D_{q_k^+} - \frac{1}{n_F} D_q. \quad (9)$$

The non-singlet functions  $D_{q_k^-}$  and  $D_{Q_k}$  obey the equations

$$\partial_\tau D_{q_k^-} = P_{qq} \otimes D_{q_k^-} \quad (10)$$

$$\partial_\tau D_{Q_k} = P_{qq} \otimes D_{Q_k}, \quad (11)$$

while the evolution of the singlet function  $D_q$  is coupled to that of the gluon function  $D_g$  as

$$\partial_\tau \begin{pmatrix} D_q \\ D_g \end{pmatrix} = \begin{pmatrix} P_{qq} & 2n_F P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} D_q \\ D_g \end{pmatrix}. \quad (12)$$

The splitting functions were calculated in Refs. [8,19]. Given the FF at some initial scale  $\mu_0$  for the quarks  $q_k$  and gluon  $g$ , Eqs. (10–12) completely determine their evolved values at some other scale  $\mu$ , to leading order in  $\alpha_s$ .

## C. Supersymmetric Equations

In a supersymmetric model (SUSY), besides the quarks and gluon one has the superpartners: squarks  $s_k$  and gluinos  $\lambda$ . In addition to the linear combinations (6) one now defines

$$D_{s_k^+} \equiv D_{s_k} + D_{\bar{s}_k} \quad (13)$$

$$D_s \equiv \sum_k D_{s_k^+} \quad (14)$$

$$D_{s_k^-} \equiv D_{s_k} - D_{\bar{s}_k} \quad (15)$$

$$D_{S_k} \equiv D_{s_k^+} - \frac{1}{n_F} D_s. \quad (16)$$

The non-singlet function  $D_{q_k^-}$  and  $D_{s_k^-}$  evolve together as do  $D_{Q_k}$  and  $D_{S_k}$

$$\partial_\tau \begin{pmatrix} D_{q_k^-} \\ D_{s_k^-} \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{sq} \\ P_{qs} & P_{ss} \end{pmatrix} \otimes \begin{pmatrix} D_{q_k^-} \\ D_{s_k^-} \end{pmatrix} \quad (17)$$

$$\partial_\tau \begin{pmatrix} D_{Q_k} \\ D_{S_k} \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{sq} \\ P_{qs} & P_{ss} \end{pmatrix} \otimes \begin{pmatrix} D_{Q_k} \\ D_{S_k} \end{pmatrix}. \quad (18)$$

The singlet functions for quarks and squarks,  $D_q$  and  $D_s$ , are coupled to the gluon and gluino functions,  $D_g$  and  $D_\lambda$ , as

$$\partial_\tau \begin{pmatrix} D_q \\ D_g \\ D_s \\ D_\lambda \end{pmatrix} = \begin{pmatrix} P_{qq} & 2n_F P_{gq} & P_{sq} & 2n_F P_{\lambda q} \\ P_{qg} & P_{gg} & P_{sg} & P_{\lambda g} \\ P_{qs} & 2n_F P_{gs} & P_{ss} & 2n_F P_{\lambda s} \\ P_{q\lambda} & P_{g\lambda} & P_{s\lambda} & P_{\lambda\lambda} \end{pmatrix} \otimes \begin{pmatrix} D_q \\ D_g \\ D_s \\ D_\lambda \end{pmatrix}. \quad (19)$$

In leading order the SUSY Eqs. (17–19) allow us to calculate the FF for all quark and squark flavours, gluons and gluinos at some scale  $\mu$  once their values at some initial scale  $\mu_0$  are known. The SUSY DGLAP equations have been given in the literature to leading order for structure functions [16,17]. Here we have presented their form for FF. It is easy to see that one just needs to transpose the matrix elements keeping the  $n_F$  factors in the same place to move from SF to FF equations. The SUSY splitting functions were calculated in Refs. [16,17].

### III. LAGUERRE ALGORITHM

#### A. Evolution Operator

In numerical studies it is better to consider the quantity  $x D(x, \tau)$ . The evolution equations for  $x D$  are the same as those for  $D$  if one multiplies the splitting functions by  $x$ . This improves numerical stability since the  $1/x$  singularity shown by splitting functions with a gluon in the final state cancels off. In general one has

$$\partial_\tau x D(x, \tau) = x P(x) \otimes x D(x, \tau), \quad (20)$$

where  $D$  is a  $d$ -dimension vector and  $P$  is a  $d \times d$  matrix whose elements are splitting functions. In the SM  $d = 1$  for Eqs. (10) and (11) while  $d = 2$  for Eq. (12). In a SUSY model  $d = 2$  for Eqs. (17) and (18) whereas  $d = 4$  for Eq. (19).

Following Furmański and Petronzio [12] one introduces the evolution operator  $E(x, \tau)$

$$x D(x, \tau) = E(x, \tau) \otimes x D(x, 0). \quad (21)$$

The evolution operator is a  $d \times d$  matrix which satisfies the following integro-differential equation

$$\dot{E}(x, \tau) = x P(x) \otimes E(x, \tau) \quad (22)$$

and the initial condition

$$E(x, 0) = \delta(1 - x). \quad (23)$$

One introduces the Laguerre expansions

$$e^{-x}P(e^{-x}) = \sum_{n=0}^{\infty} (xP)_n L_n(x) \quad (24)$$

$$E(e^{-x}) = \sum_{n=0}^{\infty} E_n L_n(x), \quad (25)$$

with  $L_n(x)$  the Laguerre polynomials of order  $n$ , which form an orthonormal base in the interval  $(0, \infty)$  with weight  $\exp -x$ . Equivalently,  $L_n(-\ln x)$  are an orthonormal base in the interval  $(0, 1)$  with unity weight. A key property of the Laguerre polynomials is their closure under convolution

$$\int_0^x dz L_n(z) L_m(x-z) = L_{n+m}(x) - L_{n+m+1}(x). \quad (26)$$

Substituting Eqs. (24) and (25) into Eq. (22), using Eq. (26) and the fact that the Laguerre polynomials form a vectorial base one gets the following system of linear equations

$$\dot{E}_n(\tau) = \sum_{m=0}^n (x\tilde{P})_{n-m} E_m(\tau), \quad (27)$$

being

$$x\tilde{P}_0 \equiv xP_0, \quad (28)$$

$$x\tilde{P}_m \equiv xP_m - xP_{m-1} \quad m \geq 1. \quad (29)$$

Laguerre expansion of the initial condition Eq. (23) translates into

$$E_n(\tau = 0) = I. \quad (30)$$

The Laguerre expansion transform an integro-differential equation into a set of ordinary differential equations. Thus there is no need to perform any intricate quadrature. One is left with an infinite number of equations  $n = 0, 1 \dots \infty$ . In practice one truncates the Laguerre expansion at some finite  $n = NLAST$ .

## B. Expansion of the Splitting Functions

The coefficients of the Laguerre expansion of a function can be quickly calculated if its Mellin transform is known. In the present work we are interested in the product

$$F(x) \equiv xP(x), \quad (31)$$

where  $P(x)$  can be any splitting function. The Laguerre series is

$$F(e^{-y}) = \sum_{n=0}^{\infty} F_n L_n(y). \quad (32)$$

The Laguerre coefficients  $F_n$  are given by

$$F_n = \int_0^{\infty} dy e^{-y} L_n(y) F(e^{-y}), \quad (33)$$

where  $L_n(y)$  is the Laguerre polynomial of order  $n$ . Let  $\hat{F}(s)$  be the Mellin transform of  $F(x)$

$$\hat{F}(s) \equiv \int_0^1 dx x^{s-1} F(x) \quad (34)$$

then one can calculate the  $F_n$ 's by means of the formula [12]

$$\frac{1}{1-u} \hat{F}\left(\frac{1}{1-u}\right) = \sum_{n=0}^{\infty} F_n u^n. \quad (35)$$

For the SM the Mellin transform of the splitting functions to leading order have been listed in many works, see for example [19]; for a SUSY model the Mellin transforms of the splitting functions to leading order are given in Ref. [16]. In both cases the Mellin transforms of  $xP(x)$  are linear combinations of only six simple functions: 1,  $1/(s+n)$  with  $n = -1, 0, 1, 2$  and  $\sigma(s) \equiv \gamma + \psi(s+1)$  where  $\gamma$  is the Euler constant and  $\psi(s)$  is the digamma function. Using Eq. (35) it is easy to show that each of these six simple functions that appear in the Mellin transform  $(x\hat{P})(s) = \hat{P}(s+1)$  gives a fixed contribution to  $F_n$ . The rules to calculate the Laguerre coefficients of  $xP(x)$  from its Mellin transform are then:

$$1 \longrightarrow 1 \quad (36)$$

$$\frac{1}{s-1} \longrightarrow \delta_{n0} \quad (37)$$

$$\frac{1}{s} \longrightarrow \left(\frac{1}{2}\right)^{n+1} \quad (38)$$

$$\frac{1}{s+1} \longrightarrow \frac{1}{2} \left(\frac{2}{3}\right)^{n+1} \quad (39)$$

$$\frac{1}{s+2} \longrightarrow \frac{1}{3} \left(\frac{3}{4}\right)^{n+1} \quad (40)$$

$$\sigma(s) \longrightarrow \left[ \left(\frac{1}{2}\right)^{n+1} + \delta_{n0} \right] - Z(n)(1 - \delta_{n0}). \quad (41)$$

As usual one defines

$$Z(n) \equiv \sum_{m=1}^n (-1)^m \binom{n}{m} \zeta(m+1), \quad (42)$$

where  $\zeta(n)$  is the Riemann  $\zeta$  function. All the Laguerre coefficients for SM and SUSY splitting functions are listed in Appendix A.

### C. Algorithm in $d$ dimensions

Let us find the recursive relations that will allow us to solve the DGLAP equations for any arbitrary dimension  $d$ . The quantities  $E_n$  and  $xP_n$  are  $d \times d$  matrices. In analogy with the  $d = 1, 2$  formulae presented in [12], let us try the following ansatz for the solution to Eq. (27):

$$E_n(\tau) = \sum_{i=1}^d e^{\lambda_i \tau} \sum_{k=0}^n \frac{\tau^k}{k!} B_{i,n}^k, \quad (43)$$

where the  $\lambda_i$ 's are the eigenvalues of  $xP_0$ . For every  $i, k, n$  with  $0 \leq k \leq n$   $B_{i,n}^k$  is a constant  $d \times d$  matrix; we have in total  $\frac{1}{2}(N_{LAST} + 1)(N_{LAST} + 2) \times d \times d^2$  matrix elements. Substituting Eq. (43) into Eq. (27) we obtain the following two matrix relations for the  $B_{i,n}^k$ ,  $d > 1$ ,



$$0 = (xP_0 - \lambda_i I) B_{i,n}^n \quad (44)$$

$$B_{i,n}^{k+1} = (xP_0 - \lambda_i I) B_{i,n}^k + \sum_{m=k}^{n-1} x\tilde{P}_{n-m} B_{i,m}^k, \quad (45)$$

while substitution of Eq. (43) into Eq. (30) gives

$$I = \sum_{i=1}^d B_{i,n}^0. \quad (46)$$

The matrix equation Eq. (44) is equivalent to  $(NLAST+1) \times d \times d(d-1)$  algebraic equations, since  $\det(xP_0 - \lambda_i I) = 0$ . Equation (45) is equivalent to  $\frac{1}{2}NLAST(NLAST+1) \times d \times d^2$  equations and Eq. (46) is equivalent to  $(NLAST+1) \times d^2$  equations. Adding up these three number we obtain  $\frac{1}{2}(NLAST+1)(NLAST+2) \times d \times d^2$  algebraic relations, which is the same as the total number of matrix elements. Therefore, the set of equations (44–46) completely determine all the matrix elements of  $B_{i,n}^k$ . They will be given as recursive relations. In order to find them one defines the projectors associated with the matrix  $xP_0$

$$\Pi_i \equiv \prod_{j \neq i} \frac{xP_0 - \lambda_j I}{\lambda_i - \lambda_j}. \quad (47)$$

They have the following properties:

$$\sum_{i=1}^d \Pi_i = I \quad (48)$$

$$\Pi_i \Pi_j = \Pi_i \delta_{ij} \quad (49)$$

$$(xP_0 - \lambda_i I) \Pi_j = (\lambda_j - \lambda_i) \Pi_j \quad (50)$$

As the key step we define suitable auxiliary matrices for  $d > 1$ :

$$b_{i,n}^0 \equiv 0 \quad (51)$$

$$b_{i,n}^k \equiv B_{i,n}^k - \sum_{j \neq i} (\lambda_j - \lambda_i)^k \Pi_j B_{i,n}^0, \quad 0 < k \leq n. \quad (52)$$

Using Eqs. (44–46) and the projector properties Eqs. (48–50) we have found that all the  $b_{i,n}^k$  and  $B_{i,n}^k$  satisfy the following relations:

$$B_{i,0}^0 = \Pi_i \quad (53)$$

$$b_{i,n}^0 = 0 \quad (54)$$

$$b_{i,n}^{k+1} = (xP_0 - \lambda_i I) b_{i,n}^k + \sum_{m=k}^{n-1} x\tilde{P}_{n-m} B_{i,m}^k \quad (55)$$

$$B_{i,n}^0 = \Pi_i + \sum_{j \neq i} \left( \frac{\Pi_i b_{j,n}^n}{(\lambda_i - \lambda_j)^n} - \frac{\Pi_j b_{i,n}^n}{(\lambda_j - \lambda_i)^n} \right) \quad (56)$$

$$B_{i,n}^{k+1} = (xP_0 - \lambda_i I) B_{i,n}^k + \sum_{m=k}^{n-1} x\tilde{P}_{n-m} B_{i,m}^k. \quad (57)$$

In the particular case  $d = 2$  these relations reduce to the ones given in [12]. The above recursive relations can easily be implemented in a computer using an object oriented language. Then we

can numerically calculate the  $B_{i,n}^k$  once the matrices  $xP_n$  are given. The evolution operator is calculated using Eq. (43) without need to integrate numerically its differential equation (27) using any finite step algorithm.

Finally, the evolved FF is given by the truncated series

$$xD(x, \tau) = \sum_{n=0}^{N_{LAST}} \sum_{m=0}^n E_{n-m}(\tau) (xD)_m (L_n(-\ln x) - L_{n+1}(-\ln x)), \quad (58)$$

where  $(xD)_m$  are the coefficients in the Laguerre expansion of the initial FF

$$xD(x, \tau = 0) = \sum_{n=0}^{\infty} (xD)_n L_n(-\ln x). \quad (59)$$

## IV. NUMERICAL ANALYSIS

### A. Evolution Steps

For clarity we will assume flavour universality in the decay of  $X$ , hence we will only consider the coupled singlet quark and gluon evolution, Eq. (12) for the Standard Model (SM), and coupled singlet quark, gluon, singlet squark and gluino evolution, Eq. (19) for a SUSY model. Particular models for  $X$  may have different branching ratios for different flavours. The source code includes all the routines necessary to evolve each quark and squark flavour.

A (s)parton is not included in the evolution as long as the energy scale is lower than its mass; when its threshold production scale is crossed, it is added to the evolution equations with an initially vanishing FF and it is assumed to be a relativistic particle.

In the SM case the code evolves the  $q$  and  $g$  initial FF from  $M_Z$  to  $M_t$ , the top quark mass, with the number of flavours set to  $n_F = 5$ , and then evolve from  $M_t$  to  $M_X$  with  $n_F = 6$ . Taking  $n_F = 6$  in the whole range from  $M_Z$  to  $M_X$  does not introduce any significant difference in the final spectrum.

In the SUSY case the code evolves the  $q$  and  $g$  initial fragmentation functions from  $M_Z$  to the supersymmetry breaking scale  $M_{\text{SUSY}} > M_t$  using the SM equations to obtain  $D_i^h(x, M_{\text{SUSY}}^2)$ , with  $i = q, g$ . Then it takes  $D_i^h(x, M_{\text{SUSY}}^2)$ ,  $i = q, g$ , and  $D_j^h(x, M_{\text{SUSY}}^2) = 0$ ,  $j = s, \lambda$ , and evolves them from  $M_{\text{SUSY}}$  to  $M_X$  using the SUSY equations. All spartons are taken to be degenerate with a common mass  $M_{\text{SUSY}}$ . In the context of structure functions a broken SUSY scenario with different masses for  $s$  and  $\lambda$  was studied [18], showing no significant difference with models with a unique SUSY mass. One expects the same result to hold for FF.

In UHECR one is interested in the final spectra of baryons  $p + n$ , neutrinos (sum of all three families) and photons. The evolution of neutrinos and photons is equivalent, from a numerical point of view, to that of baryons. In the present work we will concentrate on baryons. Initial FF are extracted from LEP data at the energy scale  $M_Z$  (see [6] for further details on initial FF and evolution of baryons, neutrinos and photons).

### B. Results

Let us first show SM evolution. In Fig. 1 we plot the fragmentation functions for baryons from quarks and gluons at the scale  $M_Z$  (fit to LEP data) and their evolved shape at  $M_X = 10^{10}, 10^{12}$

and  $10^{14}$  GeV. Following the standard convention in UHECR studies we always plot the quantity  $x^3 D_a(x, \mu^2)$ . One can see that as the final scale increases the number of particles grows at low  $x$  and diminishes at high  $x$ , a well-known result from many previous studies of scaling violations.

Next we compare SM evolution of FF with SUSY evolution. In Fig. 2 we show the common initial baryon curves at  $M_Z$ , their shape after SM evolution up to  $M_X = 10^{12}$  GeV, and their evolved shape at the same final scale after SUSY has been switched on at  $M_{\text{SUSY}} = 400$  GeV. It is clear that the SUSY curves have evolved further than the SM curves. The difference between the two scenarios stems chiefly from the different running of  $\alpha_s(\mu^2)$ . In a SUSY model  $\alpha_s$  decreases with the growth of energy scale more slowly than in the SM because of the increased contribution to the  $\beta$ -function from the SUSY partners. Since the rate of change  $\partial_{\ln \mu^2} D_a(x, \mu^2)$  is proportional to  $\alpha_s$  (see Eq. (2)), a larger  $\alpha_s$  translates into a larger amount of evolution. In other words, for the same initial and final scales we obtain  $\tau_{\text{SUSY}}(M_X) > \tau_{\text{SM}}(M_X)$ , using Eq. (5).

Figure 3 shows the quark and gluon functions at  $M_Z$ , their evolved values at  $M_X = 10^{12}$  GeV using the SUSY equations for scales larger than  $M_{\text{SUSY}}$  and the radiatively generated squark and gluino functions, all at the same final scale  $M_X$ . We find that starting from vanishing values at  $M_{\text{SUSY}}$  the squark and gluino functions start to grow and catch up with the quark and gluon functions, respectively, at small  $x$  and from scales a few orders of magnitude higher than  $M_{\text{SUSY}}$ . This behaviour can be understood qualitatively if one bears in mind that at low  $x$  the leading splitting function for quarks is  $2n_F P_{qq} \sim 4n_F C_F/x$ , which is equal to the leading splitting function for squarks  $2n_F P_{gs} \sim 4n_F C_F/x$ . For gluons and gluinos the leading splitting functions tend as well to a common value,  $P_{gg} \sim 2C_A/x$  and  $P_{g\lambda} \sim 2C_A/x$ , which is however different from that of quarks and squarks.

SUSY evolution does not depend strongly on the chosen supersymmetry breaking scale  $M_{\text{SUSY}}$ . In Fig. 3 we show the curves obtained taking  $M_{\text{SUSY}} = 200, 400$  GeV and 1 TeV. The higher the value of  $M_{\text{SUSY}}$ , the less evolved the final curves for  $q$  and  $g$ . This follows from our comparison of the SM evolution and the SUSY evolution. If SUSY switches on later (higher  $M_{\text{SUSY}}$ ) the energy range over which the SM equations hold is larger. As we have seen already, DGLAP evolution is slower when just the SM equations are employed.

### C. Convergence, Accuracy and Scope

The final FF generated by the Laguerre algorithm are accurate for large values of the evolution parameter  $\tau$ . For very small values of  $\tau$  the evolution operator approaches a delta function  $\delta(1-x)$  and thus truncation of the Laguerre series generates loss of accuracy. The final result is stable for intermediate values of  $N_{\text{LAST}}$ , the values of  $n$  at which the Laguerre expansion is truncated. In Fig. 4 we show SUSY evolution of  $q$  and  $g$  to the final scale  $M_X = 10^{12}$  GeV, for  $N_{\text{LAST}} = 9, 12, 15$ . We see that the curves approach to a common curve in most of the range of  $x$ . For large values of  $N_{\text{LAST}}$  roundoff errors start to accumulate. In double precision, for SM evolution  $N_{\text{LAST}}$  has to be smaller than 30 while for SUSY evolution, where the number of matrix operations increases substantially,  $N_{\text{LAST}}$  has to be smaller than 20.

In the limit  $x \rightarrow 0$  the Laguerre polynomials with  $n > 0$   $L_n(-\ln x)$  go to infinity. One needs a large number of polynomials to achieve good precision. However, taking  $N_{\text{LAST}}$  too large, roundoff errors become important. The code give good accuracy for  $x$  larger than a few time  $10^{-3}$ . In any case, for  $x < 10^{-3}$  the DGLAP equations (2) no longer hold [19]. For very small  $x$  gluon

emission coherence has to be taken into account; this modifies the kernel of the DGLAP equations. We have not included gluon coherence in the present version of the code.

The functions  $xD(x, \tau)$  fall rather fast when  $x \rightarrow 1$ . This fact slows down the convergence of the Laguerre series in the large  $x$  region [12]. If the asymptotic behaviour goes as  $xD(x) \sim (1-x)^\alpha$  when  $x \rightarrow 1$  then one can improve the convergence by extracting the term  $(1-x)^\alpha$  explicitly. The Laguerre series is then rewritten in terms of generalised Laguerre polynomials  $L_n^{(\alpha)}(-\ln x)$  [12]. Alternatively, one can take an analytical approach and calculate the asymptotic exponent of  $xD(x, \tau)$  in the limit  $x \rightarrow 1$  [6]. Nonetheless, one has to keep in mind that at large  $x$  FF are not well measured at low energy, hence, even in the case of perfect numerical accuracy in the evolution, the final result would not be reliable for  $x > 0.6$ .

## V. DESCRIPTION OF THE INPUT AND OUTPUT

To speed up the calculation the code takes as input the Laguerre coefficients of the initial  $xD$  at the scale  $M_Z$ . These are given in the default files `bar.l` for baryons, `gam.l` for photons and `nu.l` for neutrinos. They have been obtained from LEP data (baryons) or from the QCD Montecarlo generator HERWIG [20] (photons and neutrinos), see Ref. [6] for further details. These files contain two fields: the Laguerre coefficients for the quark singlet function and the Laguerre coefficients for the gluon function. The routine `laguerre.cc` is provided in case one wishes to start with a different set of initial data and needs to calculate the Laguerre coefficients, see Sec. VI.

The main variables of the code are set with the help of a simple user interface. There are two setup levels. The level 0 setup only allows the user who runs the code to set the final scale in the evolution  $M_X$  which is stored in the variable `double Efinal`. In this case the evolution is supersymmetric with SUSY breaking mass  $M_{\text{SUSY}} = 400$  GeV and baryons are evolved. Level 1 gives more freedom to the user. Besides the choice of  $M_X$ , now the user has to decide between SM evolution or SUSY evolution (variable `bool SUSY`), in the case of SUSY evolution  $M_{\text{SUSY}}$  must be set (variable `double SUSYMass`). The user has to select one particle to evolve: baryons, photons or neutrinos. The variable `string ParticleData` stores the name of the file with initial data for the selected particle. The standard template library class `string` is included with the header `<string>`.

If  $xD(x) \sim (1-x)^\alpha$ ,  $\alpha > 0$ , when  $x \rightarrow 1$ , the variable `alpha` can be set to an integer larger than 0 to improve convergence for large  $x$ , as explained in Sec. IV C. Level 0 defaults `alpha=0`.

There are other variables that are initialised to default values, which are the recommended values. To change them one needs to edit the main program, recompile and link. The variable `double Einit` stores the initial scale in the evolution. The default value is  $M_Z$  but it could be set to any value with  $M_b < Einit < M_t$ . Remember to recalculate the initial Laguerre coefficients if `Einit` is modified. The variable `int NLAST` stores the number of term in the Laguerre expansion ( $n = 0, 1, \dots, NLAST$ ). The variables `xmin` and `xmax` store the minimum and maximum values of  $x$ , respectively, for the output of  $xD(x)$  while `NSTEPS` controls the number of points in between.

The code evolves  $xD$  up to the final scale  $M_X$  and writes the final functions to standard output. Output has three fields for the SM ( $x$ ,  $xD_q$ ,  $xD_g$ ) and five in a SUSY model ( $x$ ,  $xD_q$ ,  $xD_g$ ,  $xD_s$ ,  $xD_\lambda$ ).

## VI. DESCRIPTION OF THE CODE

The following classes are defined in the code:

- **User, User0, User1.** They are defined in the files `User.h` and `User.cc`. They encapsulate the user interface that sets up the physical parameters of the run. The polymorphic class `User` contains the virtual function `setParameters()`, which prints a short help to standard error. The classes `User0` and `User1` inherit class `User` and redefine `setParameters()`. The function `User0::setParameters()` manages the level 0 setup while `User1::setParameters()` manages level 1.
- **StandardModel, SUSYModel.** They are defined in the files `className.h` and `className.cc`. They encapsulate the physical model. Their member functions calculate the evolution parameter  $\tau$  and the splitting functions  $P_{ba}$ . The class `SUSYModel` inherits the class `StandardModel` since at low energy any SUSY model must include the Standard Model. The class `StandardModel` inherits class `AuxFunc` (see next).
- **AuxFunc, Laguerre.** They are defined in the files `OtherClasses.h` and `OtherClasses.cc`. The class `AuxFunc` defines auxiliary functions required to calculate the Laguerre expansion of the splitting functions. In particular it stores tabulated values of the Riemann  $\zeta$  function. The class `Laguerre` encapsulates the Laguerre polynomials and the generalised Laguerre polynomials [12].
- **Vector, Matrix.** They are defined in `Arrays.h` and `Arrays.cc`. They are template classes with one template parameter which is the vector or square matrix size. The vector and matrix operators needed for the Laguerre algorithm are overloaded.
- **xD1X1, xD2X2, xD4X4.** They are defined in the header files `className.h`. These classes perform the numerical integration by calculating the matrices  $B$  using the recursive relations given in Sec. III C. The matrices  $B$  and the evolution operator  $E$  are members of these classes. Storage for the  $B$  matrices is optimised taking into account that some indices are triangular, see Eq. (43). The class `xD1X1` integrates the SM Eqs. (10) and (11), `xD2X2` integrates the SM Eq. (12) and the SUSY Eqs. (17) and (18), and `xD4X4` integrates the SUSY Eq. (19). All three classes are template classes with one generic data type which can be `StandardModel` or `SUSYModel`. All three classes inherit classes `AuxFunc` and `Laguerre`. In the present version of the code only the classes `xD2X2<StandardModel>` and `xD4X4<SUSYModel>` are used in the main program; the class `xD1X1` is provided for completeness.

The main program `Evolve.cc` takes the Laguerre coefficients of the initial FF for  $q$  and  $g$  at  $M_Z$  and calculates the final FF at  $M_X$ . It encodes the following steps:

1. It prompts the user for input and initialises variables. It declares the input file stream `qgFile` (class `ifstream` included with `<fstream>`) for the input quark singlet and gluon FF.
2. It declares the objects `sm` of type `StandardModel` and `susy` of type `SUSYModel`. The public member functions `StandardModel::setNFlavoursAndTau(EInit,EFinal)` and `SUSYModel::setTau(susyMass,EFinal)` will be called to set the number of flavours (5 below  $M_t$ , 6 above) and calculate the evolution parameter  $\tau$ .

3. It declares as well the objects `qg` of type `xD2X2<StandardModel>` and `qgs1` of type `xD4X4<SUSYModel>`. The former solves the coupled evolution of singlet quark and gluon in the SM, while the latter solves the coupled evolution of singlet quark, gluon, singlet squark and gluino in a SUSY model. The public member functions `xD2X2<StandardModel>::setB(sm)`, `xD2X2<StandardModel>::setE(sm)` calculate the matrices  $B$  and the evolution operator  $E$  in the SM whereas `xD4X4<SUSYModel>::setB(susy)`, `xD4X4<SUSYModel>::setE(susy)` calculate  $B$  and  $E$  in a SUSY model.
4. The member function `setInitialxDLaguerre()` in class `xD2X2<StandardModel>` is overloaded; it accepts an `ifstream` type argument or `void` argument. It reads the initial Laguerre expansion coefficients of  $xD(x, \tau)$  or matches Laguerre coefficients at the  $M_t$  scale. The mutator `xD4X4<SUSYModel>::setInitialxDLaguerre(qg)` matches final SM Laguerre coefficients to initial SUSY Laguerre coefficients at  $M_{SUSY}$ .
5. The public member functions `xD2X2<StandardModel>::setFinalxDLaguerre()` and `xD4X4<SUSYModel>::setFinalxDLaguerre()` calculate the evolved Laguerre coefficients. The generalised Laguerre coefficients are calculated with the member functions `setFinxDGenLag()`. Finally the accessor `getxD(xl,xr,NSTEPS,alpha)` calculates the evolved fragmentation functions  $xD(x, \tau)$  and writes the result to standard output. If `alpha=0` the Laguerre coefficients are used, if `alpha` is an integer larger than 0 the generalised Laguerre coefficients are used instead.

The header file `CommonDefs.h` sets the precision to double, defines some numerical factors and sets the maximum number of terms in the Laguerre expansion `NMAX=30`. The header file `PhysicalConst.h` defines some physical constants used in the computation.

As mentioned in Sec. V default files with Laguerre coefficients for the initial FF are provided. If one wishes to use another set of initial data, one will need to calculate their Laguerre coefficients. The file `laguerre.cc` provides a routine to calculate these coefficients when one has  $xD$  in tabulated form, i.e. one has a file with two fields: the firsts one is  $x$  and the second one  $xD(x)$ . The routine in `laguerre.cc` declares an object of class `Laguerre` and therefore must be compiled with the file `OtherClasses.cc` (the file `Makefile` is provided with the source files to facilitate compilation). Notice that the routine in `laguerre.cc` is independent of the main program in `Evolve.cc`. The Laguerre coefficients in Eq. (59) are given by

$$(xD)_n = \int_0^{\infty} dy e^{-y} (xD)(e^{-y}) L_n(y) = \int_0^1 dx xD(x) L_n(-\ln x). \quad (60)$$

The integral is performed numerically using the extended trapezoidal rule. Previous implementations of the Laguerre method tend to use parametric fits to the data of the sort  $Ax^\alpha(1-x)^\beta$  at the initial scale. In this case the Laguerre coefficients can be calculated analytically and are given in terms of infinite sums. However, the  $x$  range of validity of this fits tends to be rather narrow. Furthermore, initial data is naturally obtained in tabulated form from experiments and from Montecarlo generators. Therefore we prefer calculating the initial Laguerre coefficients as explained above without use of any intermediate parametric fit in  $x$  space.

## VII. CONCLUSIONS

In order to test models where UHECR are produced by the decay of superheavy dark matter of mass  $M_X$  one needs to calculate the predicted spectra for baryons, photons and neutrinos. The  $X$  produced injection spectrum for any primary cosmic ray is basically given by fragmentation functions at the energy scale  $M_X$ . We calculate FF by evolving low energy FF up to the ultra high energy  $M_X$  using the DGLAP equations. We have solved numerically the DGLAP equations using the Laguerre method. In our study we have considered two scenarios: Standard Model evolution and SUSY evolution. We have generalised the Laguerre method to include supersymmetry. The final result is a numerical code which is fast and accurate for  $2 \times 10^{-3} < x < 0.6$  and  $M_X > 1000$  GeV.

## ACKNOWLEDGEMENTS

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## APPENDIX A: LAGUERRE COEFFICIENTS FOR THE SPLITTING FUNCTIONS

The Laguerre expansions of the splitting functions can be calculated using Eq. (35). Let us first list the Laguerre coefficients of  $xP_{ab}(x)$  in the SM [12,14]:

$$(xP_{qq})_n = -\frac{4}{3}C_F\delta_{n0} + 2C_F \left( Z(n) + \frac{1}{4} \left( 3 - \left( \frac{1}{2} \right)^n - \left( \frac{2}{3} \right)^{n+1} \right) \right) (1 - \delta_{n0}) \quad (\text{A1})$$

$$(xP_{gq})_n = \frac{4}{3}C_F\delta_{n0} + 2C_F \left( \frac{1}{4} \left( \frac{2}{3} \right)^{n+1} - \left( \frac{1}{2} \right)^{n+1} \right) (1 - \delta_{n0}) \quad (\text{A2})$$

$$(xP_{qg})_n = T_R \left( \left( \frac{1}{2} \right)^{n+1} - \left( \frac{2}{3} \right)^{n+1} + \frac{2}{3} \left( \frac{3}{4} \right)^{n+1} \right) \quad (\text{A3})$$

$$(xP_{gg})_n = -\frac{2}{3}T_R n_F \delta_{n0} + 2C_A \left( \left( Z(n) - \left( \frac{1}{2} \right)^n + \frac{1}{2} \left( \frac{2}{3} \right)^{n+1} - \frac{1}{3} \left( \frac{3}{4} \right)^{n+1} + \frac{11}{12} \right) - \frac{2}{3}T_R N_F \right) (1 - \delta_{n0}) \quad (\text{A4})$$

Next, we present the Laguerre coefficients of  $xP_{ab}(x)$  in a SUSY model. To calculate them we use the table of Mellin transform given in [16] (in this reference there is a mismatch between the normalization of the splitting functions and the normalization of the DGLAP equations that we correct). From the rules given in Subsec. IIIB we obtain

$$(xP_{qq})_n = -\frac{11}{16}C_F\delta_{n0} + 2C_F \left[ Z(n) + \frac{1}{4} \left( 2 - \left( \frac{1}{2} \right)^n - \left( \frac{2}{3} \right)^{n+1} \right) \right] (1 - \delta_{n0}) \quad (\text{A5})$$

$$(xP_{gq})_n = \frac{4}{3}C_F\delta_{n0} + 2C_F \left[ \frac{1}{4} \left( \frac{2}{3} \right)^{n+1} - \left( \frac{1}{2} \right)^{n+1} \right] (1 - \delta_{n0}) \quad (\text{A6})$$

$$(xP_{qg})_n = T_R \left[ \left( \frac{1}{2} \right)^{n+1} - \left( \frac{2}{3} \right)^{n+1} + \frac{2}{3} \left( \frac{3}{4} \right)^{n+1} \right] \quad (\text{A7})$$

$$(xP_{gg})_n = - \left( \frac{1}{3} C_A + T_R n_F \right) \delta_{n0} + \quad (\text{A8})$$

$$\left[ 2C_A \left( Z(n) - \left( \frac{1}{2} \right)^n + \frac{1}{2} \left( \frac{2}{3} \right)^{n+1} - \frac{1}{3} \left( \frac{3}{4} \right)^{n+1} + \frac{3}{4} \right) - T_R N_F \right] (1 - \delta_{n0})$$

$$(xP_{sq})_n = C_F \frac{1}{3} \left( \frac{2}{3} \right)^n \quad (\text{A9})$$

$$(xP_{\lambda q})_n = C_F \left[ \left( \frac{1}{2} \right)^{n+1} - \frac{1}{3} \left( \frac{2}{3} \right)^n \right] \quad (\text{A10})$$

$$(xP_{sg})_n = T_R \left[ \left( \frac{2}{3} \right)^{n+1} - \frac{1}{2} \left( \frac{3}{4} \right)^n \right] \quad (\text{A11})$$

$$(xP_{\lambda g})_n = C_A \left[ \left( \frac{1}{2} \right)^{n+1} - \left( \frac{2}{3} \right)^{n+1} + \frac{1}{2} \left( \frac{3}{4} \right)^n \right] \quad (\text{A12})$$

$$(xP_{qs})_n = C_F \left( \frac{1}{2} \right)^{n+1} \quad (\text{A13})$$

$$(xP_{gs})_n = C_F \delta_{n0} - C_F \left( \frac{1}{2} \right)^n (1 - \delta_{n0}) \quad (\text{A14})$$

$$(xP_{ss})_n = -2C_F \delta_{n0} + C_F \left[ 2Z(n) - \left( \frac{1}{2} \right)^n + 1 \right] (1 - \delta_{n0}) \quad (\text{A15})$$

$$(xP_{\lambda s})_n = C_F \left( \frac{1}{2} \right)^{n+1} \quad (\text{A16})$$

$$(xP_{q\lambda})_n = T_R \left[ \left( \frac{1}{2} \right)^{n+1} - \frac{1}{3} \left( \frac{2}{3} \right)^n \right] \quad (\text{A17})$$

$$(xP_{g\lambda})_n = \frac{4}{3} C_A \delta_{n0} + C_A \left[ \frac{1}{3} \left( \frac{2}{3} \right)^n - \left( \frac{1}{2} \right)^n \right] (1 - \delta_{n0}) \quad (\text{A18})$$

$$(xP_{s\lambda})_n = T_R \frac{1}{3} \left( \frac{2}{3} \right)^n \quad (\text{A19})$$

$$(xP_{\lambda\lambda})_n = - \left( \frac{4}{3} C_A + T_R n_F \right) \delta_{n0} + \quad (\text{A20})$$

$$\left[ C_A \left( 2Z(n) - \left( \frac{1}{2} \right)^{n+1} - \frac{1}{3} \left( \frac{2}{3} \right)^n + \frac{3}{2} \right) - T_R N_F \right] (1 - \delta_{n0})$$



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# TEST RUN INPUT AND OUTPUT

% evolve

Evolve fragmentations functions up to the energy scale M\_X  
Two setup levels:  
Level 0 -> Set M\_X  
Level 1 -> Set M\_X, Standard Model or SUSY evolution,  
M\_SUSY (in the latter case), particle (baryon,  
photon or neutrino), and alpha

Enter level (0 or 1): 1  
Enter final scale M\_X in GeV: 1e12  
Enter 0 for Standard Model evolution, 1 for SUSY evolution: 1  
Enter SUSY breaking scale M\_SUSY in GeV: 400  
Enter particle (b baryons, g photons, n neutrinos): n  
Enter alpha, alpha >= 0 (type 0 if you don't know): 4

Fragmentations functions evolved up to M\_X=1e+12 GeV  
SUSY evolution with M\_SUSY=400 GeV  
Initial Laguerre coefficients read from file: ./init\_data/nu.ldat  
Parameter alpha=4

0.001 413.595 40.1029 360.98 37.8363  
0.00114574 372.335 35.3804 319.938 33.179  
0.00131271 334.828 31.14 282.839 29.0114  
0.00150402 300.755 27.3395 249.372 25.291  
0.00172321 269.822 23.9402 219.247 21.9782  
0.00197435 241.756 20.9062 192.194 19.0364  
0.00226209 216.308 18.2042 167.958 16.4314  
0.00259176 193.247 15.8036 146.305 14.1317  
0.00296947 172.365 13.6762 127.012 12.1079  
0.00340223 153.468 11.7957 109.873 10.3329  
0.00389806 136.381 10.1384 94.6951 8.78167  
0.00446615 120.945 8.68196 81.2995 7.4311  
0.00511703 107.013 7.40617 69.5181 6.25986  
0.00586277 94.454 6.29235 59.1949 5.24839  
0.00671719 83.1462 5.32339 50.1848 4.37872  
0.00769614 72.9806 4.48362 42.3533 3.63442  
0.00881775 63.8575 3.75871 35.5756 3.0005  
0.0101028 55.686 3.13559 29.7366 2.46334  
0.0115752 48.3833 2.60235 24.7301 2.0106  
0.0132621 41.8736 2.14816 20.4588 1.63113  
0.0151949 36.0872 1.76324 16.8335 1.31494  
0.0174093 30.9602 1.43872 13.773 1.05307  
0.0199465 26.4336 1.16663 11.2037 0.837566  
0.0228534 22.4526 0.939831 9.05914 0.661396  
0.026184 18.9664 0.75193 7.27963 0.518366  
0.03 15.9278 0.597254 5.812 0.40307  
0.0343721 13.2927 0.470784 4.60909 0.310818  
0.0393814 11.0197 0.368103 3.62942 0.237572  
0.0451207 9.07067 0.285354 2.83672 0.17988  
0.0516964 7.40964 0.219183 2.19954 0.134819  
0.0592305 6.00339 0.166701 1.69086 0.0999292  
0.0678626 4.82113 0.125434 1.28761 0.0731663  
0.0777527 3.83453 0.0932815 0.970304 0.052841  
0.0890841 3.01766 0.0684755 0.722594 0.0375722

0.102067 2.34699 0.0495397 0.530891 0.0262409  
0.116942 1.80132 0.0352533 0.383979 0.0179478  
0.133985 1.36174 0.0246149 0.272665 0.0119761  
0.153511 1.01153 0.0168107 0.189457 0.00775903  
0.175883 0.736047 0.0111848 0.128271 0.00485098  
0.201516 0.522559 0.00721274 0.0841796 0.00290404  
0.230884 0.360077 0.00447843 0.0531969 0.0016483  
0.264532 0.239156 0.00265422 0.0321031 0.000876103  
0.303084 0.151691 0.00148427 0.0183037 0.000429735  
0.347255 0.0907179 0.000770882 0.00972273 0.000191815  
0.397863 0.0502393 0.000363635 0.00472289 7.78663e-05  
0.455846 0.0250863 0.00015074 0.00204541 3.02731e-05  
0.52228 0.0108333 5.21486e-05 0.000762316 1.30163e-05  
0.598395 0.00376894 1.37897e-05 0.000232293 6.639e-06  
0.685603 0.000921878 2.35586e-06 5.33011e-05 3.14451e-06  
0.785521 0.000115102 1.76694e-07 7.61964e-06 8.88797e-07  
0.9 2.33062e-06 1.74687e-09 2.87264e-07 5.27823e-08

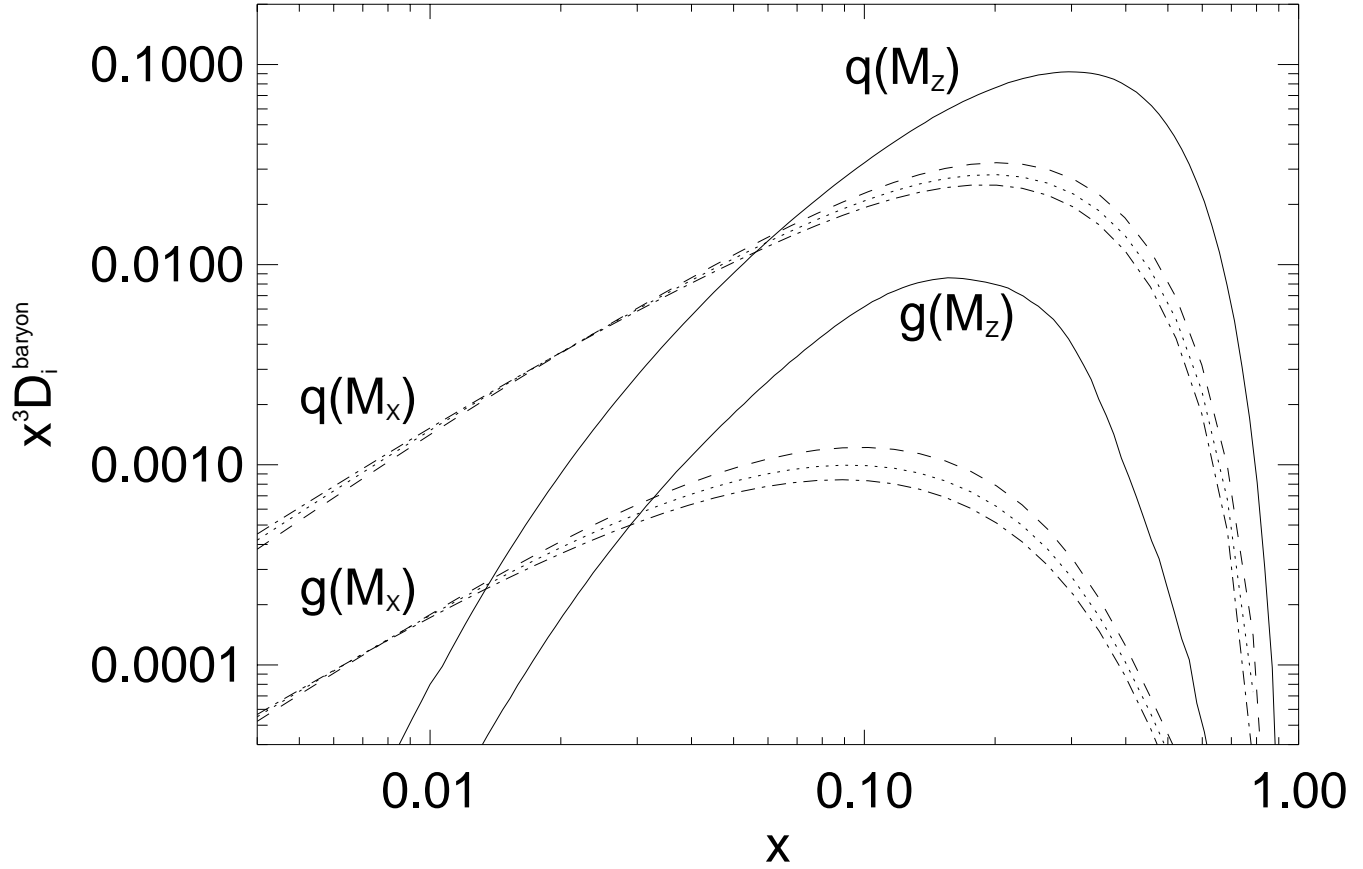


FIG. 1. Standard Model fragmentation functions for baryons from quarks and gluons, at the initial scale  $M_Z$  (solid lines) and the final scales:  $M_X = 10^{10}$  GeV (dashed line),  $M_X = 10^{12}$  GeV (dotted line) and  $M_X = 10^{14}$  GeV (dot-dashed line), showing scaling violations.

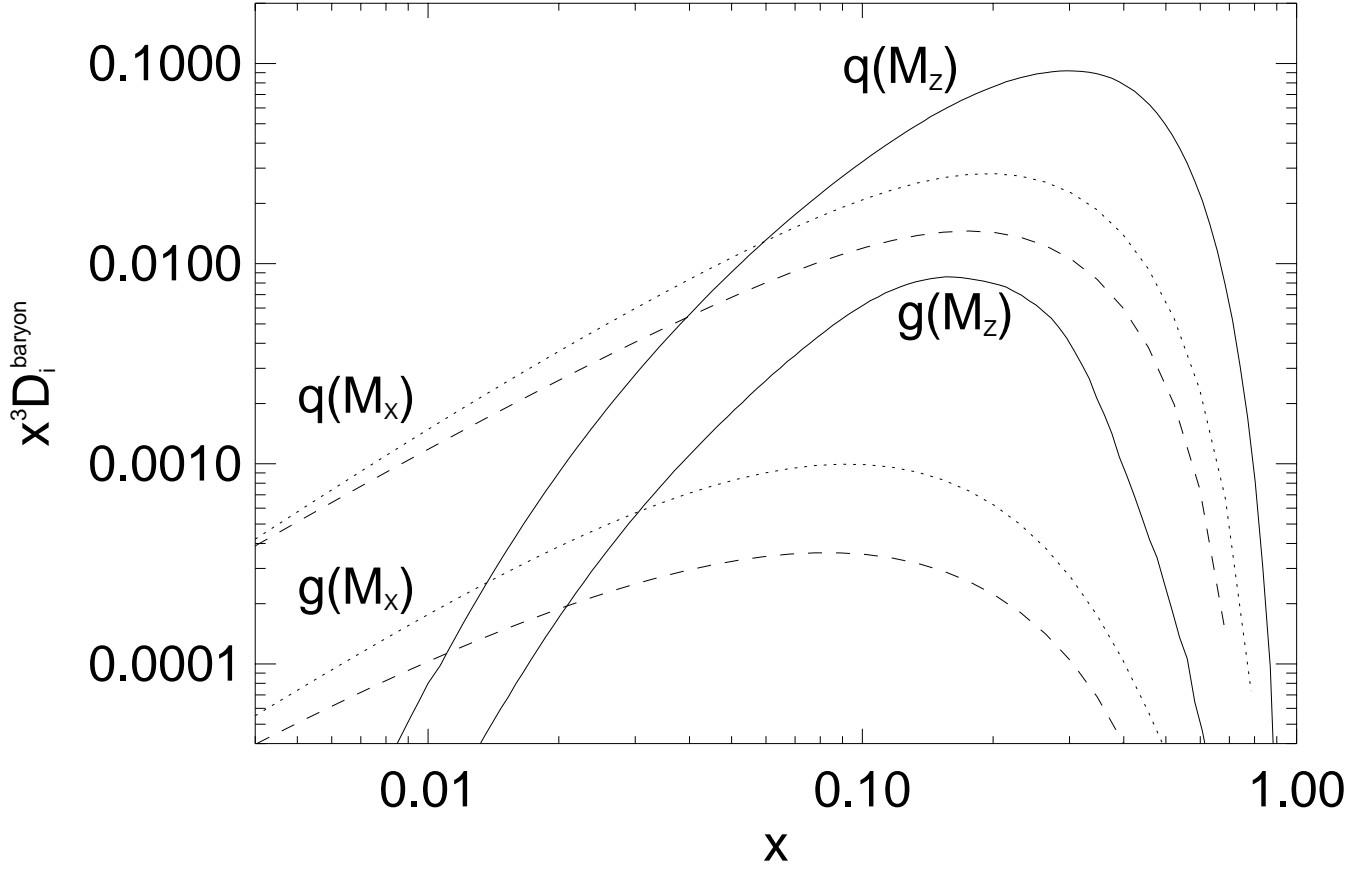


FIG. 2. Fragmentation functions for baryons from quarks and gluons, at the initial scale  $M_Z$  (solid lines) and evolved to a decaying particle mass scale of  $10^{12}$  GeV, for SM evolution (dotted lines), and, the more pronounced, SUSY evolution (dashed lines) taking  $M_{\text{SUSY}} = 400$  GeV.

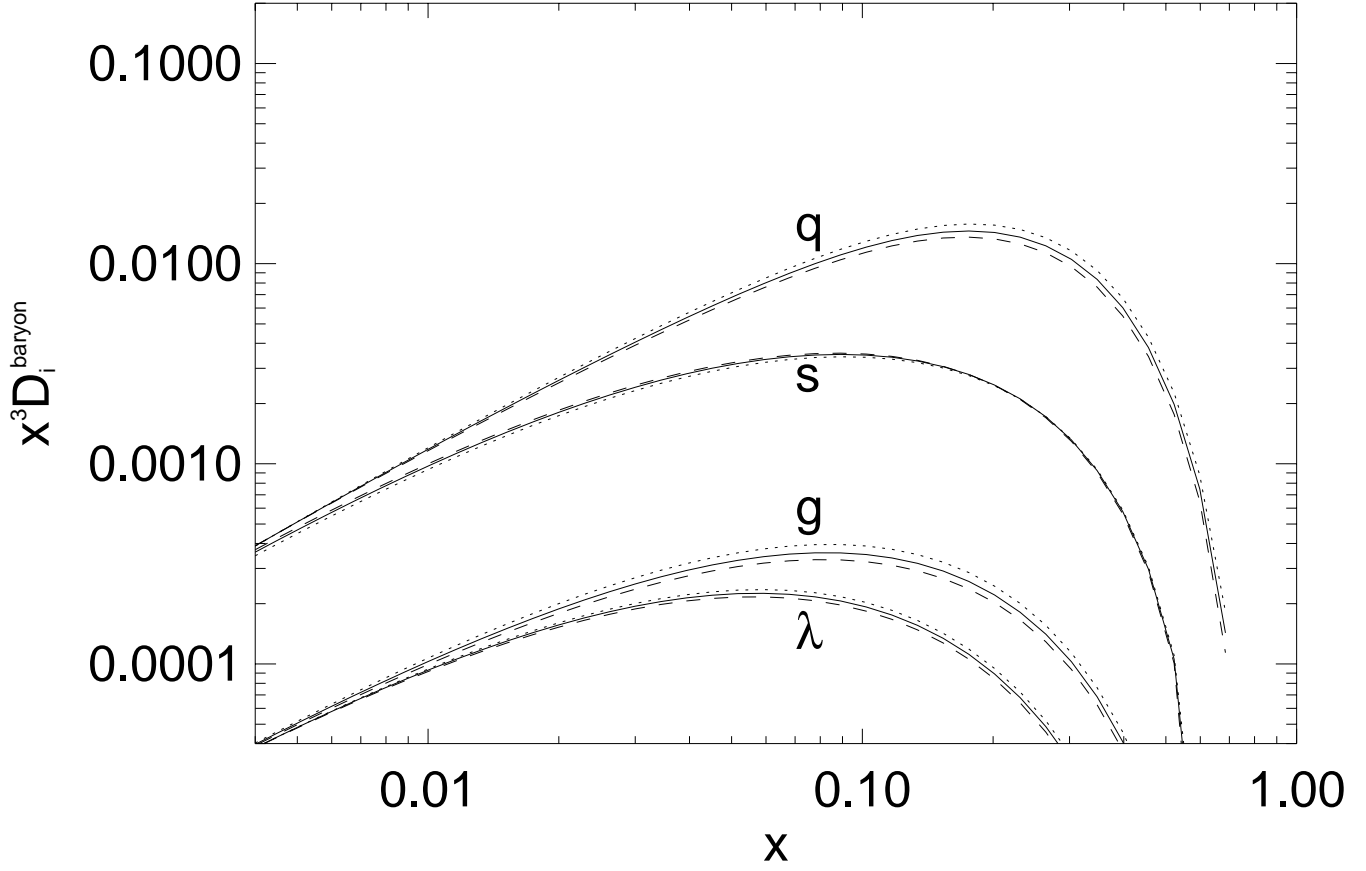


FIG. 3. Dependence on  $M_{SUSY}$ : the dashed lines are final (s)parton functions with  $M_{SUSY} = 200$  GeV, the solid lines have  $M_{SUSY} = 400$  GeV and the dotted lines have  $M_{SUSY} = 1$  TeV. The initial and final scale are always  $M_Z$  and  $M_X = 10^{12}$  GeV, respectively.

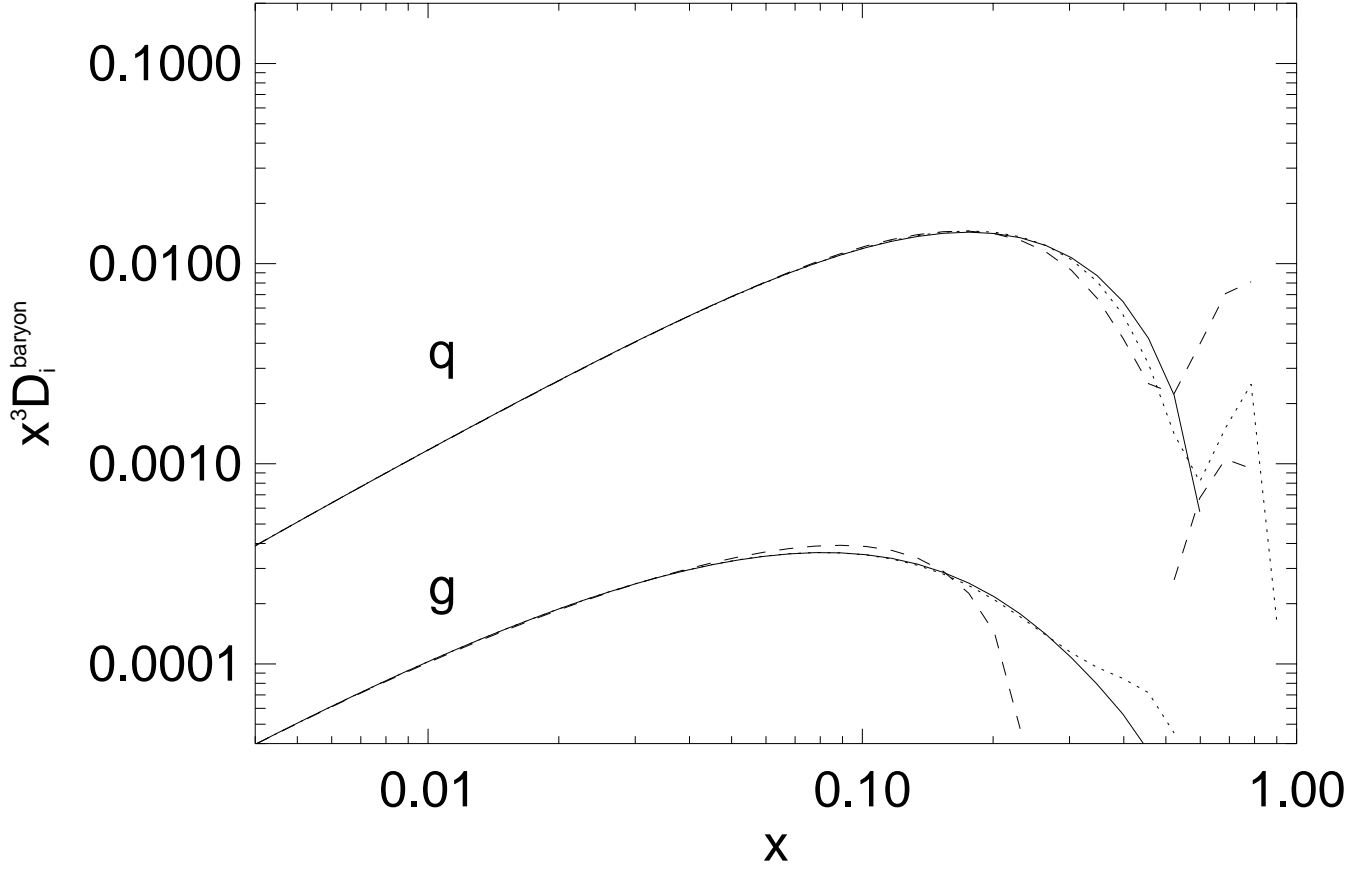


FIG. 4. SUSY evolution with  $M_X = 10^{12}$  GeV and  $M_{\text{SUSY}} = 400$  GeV. The solid lines are obtained with  $N_{\text{LAST}} = 15$ , the dot lines with  $N_{\text{LAST}} = 12$  and the dash lines with  $N_{\text{LAST}} = 9$ . All curves have been calculated with `alpha=0`. While good convergence is achieved very fast at intermediate values of  $x$ , for  $x > 0.2$  one needs to take a larger number of terms in the Laguerre expansion in order to damp out unphysical oscillations.